JURIMETRICS: THE SCIENTIFIC METHOD IN LEGAL RESEARCH*

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Introduction

The term "jurimetrics" was coined by Mr. Lee Loevinger some years ago as a convenient way of describing the use of scientific methodology in legal inquiry. Since then, its use has become fairly widespread in those esoteric circles concerned with this area of law, and it seems appropriate, therefore, to use it in the title of this article. It is convenient also to distinguish this kind of approach to law from the traditional subject of jurisprudence, by using a new term of art; although it must be said that a number of writers on jurisprudence, particularly among the so-called American legal realists, may be properly considered as being involved in the field of jurimetrics, at least in part.

Mr. Loevinger's primary concern is the development of legal research by utilizing fully the tools provided by contemporary technology, particularly digital computers. In this sense, jurimetrics may be considered a branch of cybernetics. Within the area of computer technology, Mr. Loevinger's main interest is the field of data storage and retrieval, that is, the use of computers for the

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1Jurimetrics — The Next Step Forward (1949), 33 Min. L. Rev. 455.
purpose of storing legal information (statutes, cases, articles, books, and so on) in their memories, and for referring quickly and efficiently to all relevant information when faced with a particular legal problem.

This is, however, only one use to which the computer may be put, and there are a number of others, some even more sophisticated. Moreover, it would be a mistake to adopt a limitative definition of the field of jurimetrics, as some of the potentially most fruitful areas of the application of scientific and mathematical methods to legal research do not involve computers at all, or at best indirectly. In this article, I will endeavour to sketch, very briefly, only a few of these areas as an illustration of what I conceive to be an incipient revolution in legal inquiry, which sooner or later will shake legal education, legal research and legal practice to their very foundations.

These areas are the following: Data storage and retrieval, analysis of complex evidence, prediction of judgments, legal drafting and textual ambiguities, and law reform.

I. Data Storage and Retrieval.

The tremendous potential in this area lies in increased efficiency and tremendous labour-saving. Several experiments are now in progress in the United States, some by government, others under private initiative, and of these, a number are truly operational. A number of systems do not even use computers at all, but merely employ extremely sophisticated techniques for indexing and referencing, utilizing such modern tools as micro-filming. Other systems do use computers, but important human participation is still required for the purpose of indexing of terms, formulation of problems and so on. The most advanced systems virtually eliminate the subjective processes in abstracting information for storage and in characterizing and formulating problems for solution.

2 E.g. LEX, the non-electronic indexing and microfilm storage system employed by the Antitrust Division of the United States Department of Justice.

3 E.g., Morgan’s “Point of Law” system, developed at Oklahoma State University, which is most similar to traditional approaches; and the “Keywords in Combination” technique developed by Professor J. F. Horty at the University of Pittsburgh, which is mainly useful in statutory searches. See Eldridge and Dennis, The Computer as a Tool for Legal Research (1963), 28 Law and Contemporary Problems 78.

4 Thus the Western Reserve University “Semantic Coded Abstract” system makes some use of human abstraction in indexing. The Styles “Association Factor” technique, applied to legal problems at George Washington University by Professor John C. Lyons still requires some human indexing of terms before applying probability methods. See Eldridge and Dennis, op. cit., ibid.
This is a quickly developing area, and it is hard to say what is possible. One of the most promising techniques would appear to be the entire elimination of the abstracting process. The entire text would be stored in the computer's memory, and the computer would perform a series of mathematical operations on all texts stored, determining the relative frequencies of significant terms. Problems would then be fed into the computer in as broad a way as possible, and the relative frequencies of significant terms in the problem formulation would be calculated. By performing a statistical correlation, the computer would quickly retrieve all relevant data.

The above is an all too brief description and leaves many questions unanswered. Serious problems exist, such as restricted capacities of computer memories. However, with the development of each new generation of computers, more complicated and fanciful tasks become possible, such as the accurate translation of texts written in foreign languages.

One computerized storage and retrieval system was tested against traditional methods of research in the following manner: a problem in the computer's area of competence was given to the computer and to a team of skilled lawyers, for the purpose of finding all relevant citations. The lawyers used traditional library research methods and consumed the usual wasteful amount of time before being in a position to submit the citations exhausting the point. In virtually no time at all, the computer had come up with a substantially greater number of citations. Interestingly enough, while the lawyers had missed a large number of references found by the computer, the latter had also missed a smaller number of references which were located by the lawyers! This illustrates the limitations of the computer, and its total dependency on the human beings who design and program it. Yet, on balance, the computer did a considerably more efficient job in a very much shorter time—and this efficiency can, of course, be greatly increased. The moral is obvious. Skilled, practising lawyers will, at some future date, be virtually freed of the drudgery of case-hunting. They will not need massive armies of human drudges known as juniors. Their own time will be spent in reflection; in analyzing the decided cases, not in finding them; in searching for social and other arguments to buttress their clients' positions; in preparing

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6 The American Bar Foundation and I.B.M. plan to further refine the "Association Factor" method, which might ultimately eliminate the arbitrary, human choice of indices entirely, replacing this by more reliable statistical techniques.
“Brandeis” briefs. Not only should there result substantial savings in time and expense, but lawyers’ skills will actually improve, their clients will be better served, and the courts will be enabled to render better, more reasonable judgments, in full knowledge of all relevant data. This utopia may never come altogether, but a start can be made now in Canada, as has already been done in the United States. Where computer centres exist in our universities, law faculties should insist on playing a role in their use and development, or we will not be fulfilling our duties to society and the coming generation of lawyers.

II. Analysis of Complex Evidence.

The idea here presented derives from an article by Professor Irving Kayton, of The George Washington University Law School, entitled “Can Jurimetrics be of Value to Jurisprudence?”

It is of limited application, but introduces the reader to the use of symbolic logic. Below is a contrived, hypothetical civil case, paraphrasing Professor Kayton’s example, to illustrate the possible use of modern logic as a contrast to normal non-symbolic analysis. For greatest benefit, the reader should read the problem and try to answer the four questions at the end before following the method using Boolean algebra. Those who can give the correct answers equally quickly, using ordinary verbal reasoning, are quite exceptional, according to the results of tests made by Professor Kayton. In any event, the reader should derive some increased understanding of modern logic and its possible applications from the exercise.

A boat in the Seaway crashed into a drawbridge. The only witnesses to the accident were the deck officer of the boat and the bridge operator.

To help prevent accidents, the Seaway Authority had installed an alarm system in which a warning bell is controlled by four switches, A, B, C and D.

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6 See Baade (ed.), Jurimetrics (1964), first published in (1963), 28 Law and Contemporary Problems 1-270. This symposium is a good introduction to the whole field of Jurimetrics.

7 (1964), 33 Geo. Wash. L. Rev. 287.

8 I have used similar equations and problems; Professor Kayton’s were employed in the context of a bank robbery. A slightly different type of evidentiary problem was reported on by Layman E. Allen in Law and Electronics: The Challenge of a New Era, edited by Edgar A. Jones (1962), pp. 152 to 156.

9 Boolean algebra, named for the nineteenth century British mathematician, George Boole, is the algebra used in the logical design of computers. It is the basis of modern or symbolic logic.
A is automatically turned on when the bridge is raised.
B is automatically turned on when the barriers are lowered.
C is automatically turned on when the lock is closed.
D is a switch in the bridge operator's cabin.
The bell rings if, and only if, any one of the following 3 conditions holds:
(a) The bridge is lowered and the lock is open.
(b) The barriers are down and the cabin switch is off.
(c) The lock is open and the cabin switch is on.

At trial, counsel for plaintiff (The Seaway) questioned the deck officer of the ship and the operator of the drawbridge, as well as a high official in the engineering and design department of the Seaway. All three witnesses appear equally credible. The following testimony was given:

(i) Testimony of deck officer
Q. Was the bell ringing at the time of the accident?
A. Yes.
Q. What was the condition of the bridge and the barriers at that time?
A. The bridge was raised and the barriers were raised.

(ii) Testimony of bridge operator
Q. Was the alarm sounding at the time of the accident?
A. No.
Q. What was the condition of the lock and the cabin switch at that time?
A. The lock was closed and the switch was on.

(iii) Testimony of Seaway official
He stated: "If the alarm is silent, then either the bridge is raised, or the barriers are raised, or the lock is closed."

Assuming that there is no other evidence with regard to the foregoing facts, try to answer the following questions:
(1) Was the deck officer telling the truth?
(2) Was the bridge operator telling the truth?
(3) Was the Seaway official telling the truth?
(4) Assuming the above testimony had been slightly different, so that both the deck officer and the bridge operator testified that the bridge was lowered, the barriers were raised and the lock was closed, while the deck officer said the bell was ringing and the bridge operator said it was silent, was either of them lying and if so which one?

This series of problems can, of course, be answered by means of an ordinary, non-symbolic analysis, but the mental exercise
involved is not minimal. Professor Kayton had ten subjects answer similar questions. Two persons answered all four questions correctly; five answered three questions correctly. All subjects were law professors or doctoral candidates in law. Thus, ordinary, non-symbolic analysis would appear to have certain disadvantages.

Moreover, it can be seen that in order to answer question (4) in a verbal, non-symbolic way, the problem has to be thought through again from scratch, because of the changes made in the assumptions; while, as we shall see, this is not required when using modern logic. Furthermore, there is no easy way to prove the correctness or falsity of answers based on non-symbolic reasoning; among Professor Kayton's subjects, some who were considerably in error were completely convinced that all their answers were correct. Modern logic, however, makes such proof easy, and the solutions obtained using Boolean functions are not open to question, as we shall now show. The meaning of the symbols and the necessary theorems of Boolean algebra are found in Appendix "A". Let the facts in the above case be represented by letters as follows:

\begin{align*}
X & = \text{the bell is ringing} \\
\bar{X} & = \text{the bell is silent} \\
A & = \text{the bridge is raised} \\
\bar{A} & = \text{the bridge is lowered} \\
B & = \text{the barriers are lowered} \\
\bar{B} & = \text{the barriers are raised} \\
C & = \text{the lock is closed} \\
\bar{C} & = \text{the lock is open} \\
D & = \text{the cabin switch is on} \\
\bar{D} & = \text{the cabin switch is off}
\end{align*}

Then, in Boolean notation:

\begin{align*}
(1) \quad X &= \bar{A}C + B\bar{D} + \bar{C}D \\
\text{where the + sign is used for logical disjunction (inclusive OR) and the multiplication sign (.) is used for logical conjunction (AND). By manipulation, using standard Boolean techniques as indicated in Appendix "B", we obtain, in minimal form:}
(2) \quad \bar{X} &= \bar{B}C + CD + A\bar{BD}
\end{align*}

which defines the precise conditions under which the bell is silent, just as equation (1) defines the precise conditions under which the bell rings. All four questions may now be answered by referring only to these equations, as follows.

\textbf{Question (1)} The deck officer said that the bell was ringing, the bridge was raised and the barriers were raised, that is, that \( X \) was
true and $\overline{A}\overline{B}$ was true. It can easily be shown, by manipulating equation (1), that his statements are logically consistent and thus he may be telling the truth, for

$$X = \overline{A}C + B\overline{D} + CD$$

$$= \overline{A}C + B\overline{D} + ABCD + AB\overline{C}D + \overline{ABC}D + \overline{ABC}D$$

(by Theorems 1a, 4 & 9)

and the third term satisfies his statement.

**Question (2)** The bridge operator said that the bell was silent, the lock was closed and the switch in his cabin was on, that is, that $X$ was true and $CD$ was true. These statements are consistent with the second term on the right hand side of equation (2), and therefore he may also have been telling the truth.

Note moreover that the bridge operator's statement $CD$ is inconsistent with the deck officer's statement $X$, as can be seen from equation (1). However, the deck officer's version, $AB$ is consistent with the bridge operator's statement $\overline{X}$, as appears from equation (2). Consequently, we may make the hypothesis that it is more likely, perhaps, that the bell did not ring, and that the deck officer imagined he heard it or is lying.

**Question (3)** The Seaway official asserted that if $\overline{X}$ is true then $A + B + C$ is true, that is, either the bridge is raised OR the barriers are raised OR the lock is closed. From equation (2), it can be seen that this statement is logically consistent, as every term contains $A$ or $\overline{B}$ or $C$. Therefore he was telling the truth. Similarly, had he said "If the bell is silent, then the barriers are raised or the lock is closed", that is, $\overline{X}$ implies $\overline{B} + C$, he would also have been telling the truth.

**Question (4)** In this question we assumed that both witnesses testified that $\overline{A}\overline{B}C$ was true, that is, that the bridge was lowered AND the barriers were raised AND the lock was closed. It is clear from equation (1) that this is inconsistent with $X$, while from equation (2) it can be shown to be consistent with $\overline{X}$, for

$$\overline{X} = \overline{B}C + CD + A\overline{B}D$$

$$= \overline{ABC} + ABC + CD + A\overline{B}D$$

(by Theorems 1a, 4, and 9)

and the first term on the right hand side is now $\overline{A}\overline{B}C$. Thus the deck officer's version $X$ is inconsistent with his statement $\overline{A}\overline{B}C$, while the bridge operator's version, $\overline{X}$, is consistent. But we cannot say that the deck officer is lying, for he may have imagined, in good faith, that he heard the bell. In fact, the bridge operator's internal consistency may be a tissue of lies, and the bell may have rung after all, although the deck officer made an error as to one of the other facts. Logic can only go so far; it can detect consistent and
inconsistent statements, but it cannot distinguish truth from falsehood, although it is an invaluable aid in attempting to do so. We have now gone as far as is possible in answering the four questions asked.

For those who still doubt the validity of the techniques used, there follows a "Truth Table", which gives the Boolean functional relationships in tabular form and confirms the results obtained above. A disadvantage of expressing algebraic relationships in this way is that the table become unmanageable very rapidly as the number of variables increases.

\[ X = \overline{A}C + \overline{X} = CD + \overline{ABD} + \overline{BC} \]

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Note that, in accordance with the notation in Appendix "A", the symbol \( 0 = \text{false} \), \( 1 = \text{true} \), and the value of any function \( X \), \( \overline{X} \), \( A, \overline{A}, B, \overline{B}, C, \overline{C}, D, \overline{D} \), or any combination of these using the operations \text{INVERSION}, \text{AND}, and \text{OR}, must be either 0 or 1. The rules for performing the operations \text{OR} and \text{AND} are found in Appendix "A", and it is a simple matter to verify the above table. It is always true that if \( A = 0 \), then \( \overline{A} = 1 \), and \textit{vice versa}.

III. Prediction of Judgments.

The prediction of judgments promises to be one of the most important areas in which mathematical techniques and the behavioural sciences can assist the legal scholar and practitioner. The
use of mathematical models greatly increases the accuracy of prediction, although, of course, no certainty can ever exist in this area. Still, a substantial improvement in lawyers' skills is most significant. The models discussed below can be divided into two groups: (a) behavioural models; and (b) stare decisis models.

(a) Behavioural models.

This type of model is perhaps the most important, as where it is feasible to construct such a model, a high degree of accuracy in prediction is possible, whether one is concerned with a rapidly changing and varying area of law, or one in which a relative degree of stability exists.

Thus, in the case of the Supreme Court of the United States, Professor Glendon Schubert\(^\text{10}\) and others have been able to use advanced statistical techniques including factor analysis, to accomplish a multitude of tasks. Firstly, the existence of "blocs" among the court's members was clearly established on a mathematical basis, in terms of the frequencies with which the Justices tended to vote with each other. This kind of analysis is purely mathematical and does not involve typing any judge as "liberal" or "conservative". Secondly, by a suitable choice of factors, and an analysis of past voting behaviour of the judges in various types of cases (irrespective of the reasons given by the judges for their votes), a three-dimensional pattern was established for the purpose of predicting how the court would vote in the series of legislative reapportionment cases decided in 1964. The prediction was that reapportionment would be ordered in all the cases considered, and that Justices Douglas, Black, Goldberg, Warren and Brennan would always be in the majority, while Justice Harlan would dissent; moreover, that where any of the remaining three Justices joined the majority they would do so in the sequence: White, Clark, Stewart. This prediction was entirely fulfilled; in fact, Mr. Justice White sided with the majority in all cases.\(^\text{11}\)

Now it may be argued that a skilled lawyer could have made the same kind of predictions without recourse to statistics or factor analysis, and Professor Fred Rodell of Yale has been cited as an

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\(^{10}\) Judicial Attitudes and Voting Behaviour: The 1961 Term of the United States Supreme Court (1963), 28 Law and Contemporary Problems 100.

\(^{11}\) In *Beadle v. Scholle* (1964), 377 U.S. 990, *certiorari* was denied as Professor Schubert had predicted; in *Reynolds v. Sims* (1964), 377 U.S. 533, the vote was 8 to 1; in *W.M.C.A. v. Simon* (1964), 377 U.S. 633 it was 6-3; and so on down the line. The prediction held true for cases not docketed at the time of writing.
example of the kind of successful prediction that is possible on an intuitive basis. The least that can be said is that intuitive prediction is non-replicative, while mathematical prediction is. It is submitted that the statistical approach, which is capable of further refinement and development, can be of great service. It is interesting to note that the laborious mathematical computations required by this method would be impossible without the assistance of computers to do the arithmetic.

You may be interested to know that a beginning has been made in Canada along the above lines, following in the footsteps of Professor Schubert. The Royal Commission on Bilingualism and Biculturalism has underwritten a research project by Professor Peter Russell, a political scientist at the University of Toronto, on the Supreme Court of Canada. Professor Harry Arthurs, of the Osgoode Hall Law School, is acting as legal consultant to the project. At the time of writing, all relevant data on the approximately 1100 reported cases decided by the court since 1949 were being fed into the computer, and we should know fairly soon whether in fact, the Supreme Court of Canada divides into blocs, in the same way as its sister tribunal to the south. It should be extremely interesting to follow this research, and especially if the blocs are proved to exist, to determine if some basis of prediction in selected areas of law is possible in Canada, as it is in the United States, on a behavioural basis. This would be most significant, as we have always been told that true stare decisis plays a far greater role here, and that considerations of policy or the personality of the individual judge are far less important here than in the American judicial system.

(b) **Stare decisis models.**

The utility of these models is more restricted, as they can only tell us what a court will do if it follows so-called pre-existing rules of law. This obviously has greater value in relatively stable areas of law. However, it is also most useful to know what the law *has purported to be* even where high predictability does not exist, as complex rules may be made much simpler using mathematical models, and in any event, what the rules purport to say up to a particular case will always be one of the important factors in deciding that case, even if these rules are not decisive. I will mention only two such models.\(^\text{12}\)

\(^{12}\) See Kort, Simultaneous Equations and Boolean Algebra in the Analysis of Judicial Decisions (1963), 28 Law and Contemporary Problems 143, from which the following is taken.
tions, and its utility is questionable. The number of unknowns is equal to the number of possible relevant facts in each case; the coefficient of each unknown is 0 or 1, depending on whether that fact is present or absent in that particular case; and the sum of the unknowns, each multiplied by its appropriate coefficient, is equal to the number of judges rendering the desired decision (out of the total Supreme Court Bench). In three decided cases dealing with the same problem we could postulate the following results:

\[
\begin{align*}
    x_1 + x_2 + x_3 &= 7 \\
    x_1 + x_3 &= 5 \\
    x_1 + x_2 &= 3
\end{align*}
\]

This indicates that in case 1, all three facts \( x_1, x_2, \) and \( x_3 \) were present, and the judges voted 7 - 2; in case 2, only facts \( x_1 \) and \( x_3 \) were present, and the judges voted 5 - 4; in case 3, only facts \( x_1 \) and \( x_2 \) were present, and judges voted 3 - 6, that is, the desired result was not obtained. Solving these three equations, we get \( x_1 = 1, x_2 = 2, x_3 = 4 \). We now know the respective weights to be given to these facts in future cases, and we can predict, for instance, that if facts \( x_2 \) and \( x_3 \) are present, the vote will be 6 - 3. This model's usefulness is restricted by its underlying assumptions. Nonetheless, it is possible to use it in situations involving very large numbers of facts, again by employing factor analysis as well as the computer for the laborious computations involved.

(ii) A more exciting model is that using symbolic logic, that is, Boolean algebra, which has been discussed in detail above. This method reduces to a mathematical formulation the precise logical rule that the courts have employed in the past in a given area.

In the "right to counsel" cases, Mr. Reed Lawlor devised the following formulation for the rule of Betts v. Brady,\(^{13}\) prior to the United States Supreme Court decision in Gideon v. Wainwright:\(^{14}\)

\[
    X = (f_{11} + f_{12}) \cdot f_{19} \cdot [L(1, S_a) + L(5, S_b)]\(^{15}\)
\]

where \( X \) indicates a decision that the right to counsel exists; \( f_{11} \), that the petitioner had no assistance of counsel at arraignment; \( f_{12} \), that petitioner had no assistance of counsel between arraignment and trial or plea of guilty; \( f_{19} \), that petitioner had not waived explicitly the right to counsel; subset \( S_a \) contains two facts and subset \( S_b \) contains thirty-two facts. This expression means that the petitioner will succeed if (and only if) \( f_{11} \) or \( f_{12} \) or both, and \( f_{19} \) exist, and at least one element of subset \( S_a \) or at least five elements

\(^{13}(1942), 316 U.S. 455.\)
\(^{14}(1963), 372 U.S. 335.\)
\(^{15}\)Lawlor developed a similar equation in Foundations of Legal Decision Making, [1963] Modern Uses of Logic in Law 98 (Generally referred to as M.U.L.L.).
of $S_b$ or both are present. Note the advantage of using set notation for $S_b$ as the ordinary Boolean notation would have required a total of $32C_5 = 201,376$ combinations of five facts each connected by $+$ signs (disjunctions). Obviously, the computer is essential here to solve the logical equation and identify any relevant subset of elements.

The obvious difficulty with this type of equation is, of course, that the United States Supreme Court proceeded to reverse Betts v. Brady in Gideon v. Wainwright, ignoring the above equation and *stare decisis*. Nonetheless, the use of Boolean algebra can greatly simplify the formulation of complex legal rules and can at least tell you what a court will do if it follows *stare decisis*. Moreover the court itself will be in a position to state whether or not it is making new law, resulting in greater clarity and meaningfulness in judicial reasoning, and consequently greater accuracy in prediction by lawyers, who will see more clearly the real reasons of a court, with the mask of *stare decisis* removed. This would be completely in accordance with the wishes of the American legal realists.

It is suggested that in view of the new mathematical and statistical techniques in legal prediction, any legal education in the future should be considered incomplete, unless it has included at some stage—perhaps pre-law—a study of set theory, probability, statistics (perhaps including factor analysis), symbolic logic and Boolean algebra. For the legal researcher, these tools will be essential; the practitioner will at least need the results of their employment, even if he is unable to use them himself; and he will be a worse practitioner if he lacks this ability.

IV. Legal Drafting and Textual Ambiguities.

Symbolic logic and Boolean algebra can play a vital role in the analysis and drafting of legal documents of all kinds, including statutes, contracts, wills, treaties, and so on. A great deal has been written on the subject in the periodical *Modern Uses of Logic in Law*, a joint project of the Yale University Law School and the American Bar Association. *Modern Uses of Logic in Law* is specialized in jurimetrics and has published extensive material on other uses of symbolic logic in law, as well as the various applications of computers, including data storage and retrieval. In an article in the *American Behavioural Scientist*, Professor Layman E.

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16 (1963), 7 Amer. Beh. Sc. 39, at p. 41. The chart on that page contains some clerical errors, however: Interpretation (1) is really Interpretation
Allen, editor of *Modern Uses of Logic in Law*, has done an interesting analysis of the Nuclear Test Ban Treaty, using an adaptation of symbolic logic for lawyers, including the traditional truth-table. It appears quite clearly from his analysis—and anyone can duplicate it—that because of poor draftsmanship there may now be certain kinds of tests which could be considered permissible or prohibited, with equal logic. Professor Allen’s symbolism is a compromise with traditional legal techniques, and is designed to eliminate the need for a knowledge of standard symbolic logic methods. This may achieve the desired goal of reducing lawyer resistance, and may also make the results slightly more accessible, but a number of serious disadvantages exist, such as the fact that rigorous analysis, including the use of inverse functions, becomes much more difficult, if not impossible. Let us see how the analysis looks, using standard Boolean algebra methods.

Article I of the Treaty commences as follows:

1. Each of the Parties to this Treaty undertakes to prohibit, to prevent, and not to carry out any nuclear weapon test explosion, or any other nuclear explosion, at any place under its jurisdiction or control: (a) in the atmosphere; beyond its limits, including outer space; or underwater, including territorial waters or high seas; or (b) in any other environment if such explosion causes radioactive debris to be present outside the territorial limits of the state under whose jurisdiction or control such explosion is conducted. . . .

If we let $X$ = nuclear explosions to be prevented or prohibited; $A$ = nuclear weapon test explosion; $\bar{A}$ = any other nuclear explosion; $B$ = at any place under the testing state’s jurisdiction or control; and $C$ = everything bracketed in subparagraphs (a) and (b), then the following possible interpretations of paragraph 1 of article I exist and are equally logical:

$$X_1 = ABC + \bar{A}BC$$
$$X_2 = AC + \bar{A}BC$$
$$X_3 = A + \bar{A}BC$$

Using standard techniques, we can simplify these equations, and find their inverses, with the following results:

*Interpretation (1)*  
$X_1 = BC$  
$\bar{X}_1 = \bar{B} + \bar{C}$

(3), Interpretation (2) is Interpretation (1), and Interpretation (3) is actually Interpretation (2).

Interpretation (2)  \[ X_2 = AC + ABC \]
\[ \overline{X}_2 = \overline{A}B + C \]

Interpretation (3)  \[ X_3 = A + BC \]
\[ \overline{X}_3 = \overline{A}B + \overline{AC} \]

Under interpretation (1), explosions are only to be prevented and prohibited if on the state’s territory and coming under condition C. They need not be prevented or prohibited if they are either outside the state’s territory, or they do not come under C. Underground weapons tests anywhere which do not contaminate areas outside the state’s territory need not be prevented by it. Furthermore underground weapons tests which do not contaminate outside areas need not be prevented or prohibited if conducted outside the state’s territory, for instance, at the South Pole. This interpretation appears to be that intended by the contracting states, that is, no distinction between weapons tests and other nuclear explosions. It is the narrowest possible interpretation of the treaty, as it calls on states to prohibit only two out of eight kinds of explosions. This can be seen from the following truth table:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>X_1</th>
<th>\overline{X}_1</th>
<th>X_2</th>
<th>\overline{X}_2</th>
<th>X_3</th>
<th>\overline{X}_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tbody>
</table>

Under interpretation (2), explosions are to be prevented and prohibited only if they are weapons tests and come under C, or if they are on the state’s territory and come under C. They may be permitted in all cases if they do not come under C, or if they are not weapons tests and occur outside the state’s territory. Underground explosions at the South Pole which contaminate other countries are permissible as in interpretation (1). So are underground weapons tests conducted anywhere which do not contaminate others. This does not seem to make sense. Interpretation (2) is wider than (1), as it obliges states to prevent and prohibit three kinds of explosion out of eight, as the above truth table shows.

18 The same applies to weapons tests in the non-territorial air space, or on the high seas, or in outer space.
Finally, interpretation (3) requires states to prevent and prohibit all weapons tests, and all explosions on the state's territory which come under C. It permits explosions which are not weapons tests, and which are either outside the state's territory or do not come under C. Underground explosions (other than weapons tests) at the South Pole are permitted even if they contaminate, while underground explosions which are not weapons tests on the state's territory can be permitted only if they do not spread contamination outside the state. Again, this does not seem to make sense. This broadest interpretation of the treaty prohibits five out of eight kinds of explosions.

The above alternative interpretations of the treaty can be illustrated diagrammatically by means of computer circuits, where the logic blocks “OR”, “AND”, and “INVERTER”, represented by $+$, $\cdot$, and $\bar{}$, are simple electronic devices performing the same operations as those of true-false logic. If we consider the inputs into such a logic block and its output as wires in which current is or is not flowing, and if we use the symbol “0” for no current and “1” for current, then the blocks are defined as follows.

(a) The “OR” block has an output of “1” if at least one input is equal to “1”:

$$\begin{align*}
0 & \quad \bar{} \\
0 & \quad + \\
1 & \quad + \\
1 & \quad + \\
\end{align*}$$

(b) The “AND” block has an output of 1 only if all inputs are equal to 1:

$$\begin{align*}
0 & \quad \bar{} \\
0 & \quad \cdot \\
1 & \quad \cdot \\
1 & \quad \cdot \\
\end{align*}$$

(c) The “INVERTER” block has an output of 1 if the input is 0, and an output of 0 if the input is 1:

$$\begin{align*}
A & \quad \bar{} \\
A & \quad \bar{} \\
0 & \quad \bar{} \\
1 & \quad \bar{} \\
\end{align*}$$

The “OR” or “AND” blocks may have any number of inputs, but only one output. The “INVERTER” block has exactly one input and one output.

The circuits illustrating the 3 interpretations of the treaty are then the following:

---

19 Also permitted are explosions which are not weapons tests conducted outside the state's territory in the atmosphere, on the high seas, or in outer space.

20 This interpretation is the least plausible because the physical layout of paragraph 1 of article I implies that C applies to all explosions, whether weapons tests or not.

21 “0” and “1” may merely indicate low or high voltage in the circuit.
These schematic circuits are clear diagrammatic representations of the three interpretations. They again indicate how, in truth, Boolean algebra is the foundation of computer technology.

From the above analysis of the treaty, it can be seen how syntactic ambiguity can result in legal problems of immense consequence. Thus, if the Soviet Union or the United States of America decides to permit a certain type of explosion, and the other says that this is a violation of the treaty, the problem cannot be solved by reference to the logical meaning of the words alone. Any solution would have to take into account the intention of the parties, what is said elsewhere in the treaty, and so on. Of course, the ambiguities in paragraph 1 of article I really only affect explosions caused by private persons or by states other than the contracting parties themselves, which explosions the contracting parties undertake to prohibit and prevent. Paragraph 2 of the same article obliges the contracting parties themselves to refrain from participation in tests, in the following words:

2. Each of the Parties to this Treaty undertakes furthermore to refrain from causing, encouraging, or in any way participating in, the carrying out of any nuclear weapon test explosion, or any other nuclear explosion, anywhere which would take place in any of the environments described, or have the effect referred to, in paragraph 1 of this Article.

If we let

- \( Y = \) prohibited nuclear explosions
- \( A = \) nuclear weapon test explosion
- \( \bar{A} = \) other nuclear explosion
- \( C = \) everything bracketed in subparagraphs (a) and (b) of article I, paragraph 1,

then the following two possible interpretations of paragraph 2 exist:

\[
Y_1 = AC + \bar{A}C = C \quad \text{and} \quad \bar{Y}_1 = \bar{C}
\]

or

\[
Y_2 = A + \bar{A}C = A + C \quad \text{and} \quad \bar{Y}_2 = \bar{AC}
\]
Of these, the $Y_1$ formulation is more plausible. But under the $Y_2$ formulation, a contracting state could not itself participate in a nuclear weapon test explosion, even underground and with no escape of radioactive dust. $Y_2$ is therefore more restrictive, as under $Y_1$, any nuclear test may be conducted by a contracting state if underground and non-contaminating in character.

Assuming that interpretations $X_1 = BC$ and $Y_1 = C$ were intended by the contracting parties, paragraphs 1 and 2 can now be rewritten as follows, without risk of ambiguity:

1. Each of the Parties to this Treaty undertakes to prohibit and prevent any nuclear explosion at any place under its jurisdiction and control:
   (a) in the atmosphere or beyond its limits
   (b) underwater, including territorial waters
   (c) in any other environment, ....

2. Each of the Parties to this Treaty undertakes furthermore to refrain from causing, encouraging, or in any way participating in, the carrying out of any nuclear explosion:
   (a) in the atmosphere; beyond its limits, including outer space; or underwater, including territorial waters or high seas; or
   (b) in any other environment ....

This new wording places weapons tests and explosions for the purpose of digging canals on exactly the same footing. Had the great powers employed draftsmen skilled in symbolic logic to frame the treaty, the problems created by syntactic ambiguity could easily have been avoided.22

Many statutes and contracts are similarly ambiguously worded, and these ambiguities may lead to problems at later stages of litigation which could be avoided. Semantic ambiguities may at times be desirable, to permit flexibility, but syntactic ambiguities are mere failures to express a clear intention which then disappears, leaving the later interpreter a choice between competing logically consistent interpretations, one of which is patently false.

As has been said above, the language of the computer is the true-false logic of Boolean algebra. Training in computer technology is a means of acquiring the ability to draft in unambiguous language; for in the design of computer circuits or in the formulation of instructions to the computer, ambiguity must be eliminated. It has even been suggested that statutes should always be worded in a way that lends itself to computer design and programming. This would facilitate the integration of law and computer tech-

22 Note also the imprecise drafting in paragraph 1, in that outer space and the high seas are subsumed under a “place under its jurisdiction and control”. This also becomes apparent as a result of the above analysis.
ology for all purposes, including storage and retrieval of information, and the solution of logical problems.

For example, section 2 of the Income Tax Act\textsuperscript{23} now reads as follows:

(1) **Residents.** An income tax shall be paid as hereinafter required upon the taxable income for each taxation year of every person resident in Canada at any time in the year.

(2) **Non-residents employed or carrying on business in Canada.** Where a person who is not taxable under subsection (1) for a taxation year
   (a) was employed in Canada at any time in the year, or
   (b) carried on business in Canada at any time in the year,
   an income tax shall be paid as hereinafter required upon his taxable income earned in Canada for the year determined in accordance with Division D.

(3) **Taxable Income.** The taxable income of a taxpayer for a taxation year is his income for the year minus the deductions permitted by Division C.

If we wished to design a computer circuit to determine when a tax is payable under section 2, we would have to reword subsections (1) and (2) more or less as follows:

An income tax shall be paid \textsuperscript{Iff} a person is resident in Canada

\[
X = AB + D(C + E) = AB + CD + DE
\]

Note that again the multiplication sign is used here for logical conjunction (\&), the + sign for logical disjunction (\lor, the inclusive OR), and \overline{A} is used for the inverse of A(\overline{A}, A', "NOT A").

\textsuperscript{23} R.S.C., 1952, c. 148, as am.
\textsuperscript{24} The symbol IFF is used here to designate "if and only if".
computer circuit can then be designed here using only the "AND" and "OR" logic blocks, each of which is a simple diode device, the inputs being A, B, C, D, and E, and the output being X. When needed, we may obtain \( \bar{A} \) by using an inverter block, which is merely a transistor device. Thus for \( X = AB + D(C + E) = AB + CD + DE \) we have the alternative circuits

![Diagram of computer circuit with inputs A, B, C, D, E, and output X.]

This is, of course, a trivial example, but the same method can be adapted to far more complex situations. Even this example illustrates the greater simplicity of computer language, which requires that a legal rule be framed in terms of symbolic logic. If this type of language were widely adopted, it would make for greater uniformity in legal phraseology, easier understanding by both lawyers and laymen, and much simpler adaptability to computers for problem solution or information storage, as well as eliminating all syntactic ambiguities. Even the exercise in changing the language of a statute to computer form results in the detection of all syntactic ambiguities, as the nuclear test ban example illustrates.

Here is another example to show how syntactic ambiguities can be detected by this method. Article 15 of the Quebec Code of Civil Procedure\(^{25}\) reads, in part, as follows:

The courts cannot sit between the thirtieth day of June and the first day of September, and in addition they are not obliged to sit between the thirty-first day of August and the tenth day of September, or between the twentieth day of December and the tenth day of January, except, in either case, as regards: [there follows a list of special cases].

Rewording the first paragraph, we get, alternatively:

(a) (i) The courts cannot sit IFF

\[ X \]

it is between June 30 and September 1 and

\[ A \]

\(^{25}\) This Code, which became law in 1897, will be shortly replaced by a new Code, S.Q., 1965, c. 80.
one of the enumerated cases does not hold:

\[ B \]

(ii) The courts are not obliged to sit \( \text{IFF} \)

\[ Y \]

it is between August 31 and September 10 and

\[ C \]

one of the enumerated cases does not hold, or

\[ B \]

it is between December 20 and January 10, and

\[ D \]

one of the enumerated cases does not hold

\[ B \]

\( \text{i.e.} \) (i) \( X = AB \)

(ii) \( Y = (C + D)B = BC + BD \)

or we get, with equal logic:

(b) (i) The courts cannot sit \( \text{IFF} \)

\[ X \]

it is between June 30 and September 1:

\[ A \]

(ii) (remains unchanged)

\( \text{i.e.} \) \( X = A \)

\( Y = BC + BD \)

Obviously (a) represents the intention of the legislature, and this is substantiated by the French version of the article, which reads:

Les tribunaux ne peuvent siéger entre le trente juin et le premier septembre, et, en outre, ne sont pas tenus de siéger entre le trente et un août et le dix septembre, ni entre le vingt décembre et le dix janvier, excepté dans chacun de ces cas, lorsqu'il s'agit: . . . .

However, better draftsmanship would have eliminated the ambiguity immediately.

V. Law Reform.

I wish here to give only a few examples of the kind of research that can and should be pursued in this area. The possibilities of inter-
disciplinary collaboration in research have barely begun to be explored. Penology has received perhaps the most attention in the past. One area that can and should be investigated immediately on an inter-disciplinary basis is that of the law of evidence and the whole process of fact-finding and trial. The psychologists and other behavioural scientists can be of immense help to us in this area. Our antiquated rules governing trial procedure and evidence have a large measure of common sense in them, but a still larger measure of traditionalism. The rules (and rules-of-thumb) used by judges to evaluate the credibility of witnesses need a thorough investigation, and their fallacies need to be exposed. The purpose of a trial is to find the truth, and technical rules which hinder this process are worse than useless—they are a source of injustice. We must endeavour to determine scientifically the criteria which should be used to evaluate credibility, techniques for detecting unreliability or dishonesty, poor memory, prejudice, bias, or outright lying. We must be able to answer questions such as whether the direct evidence of an eye-witness is always the best evidence or whether contradictions in the testimony of a witness are necessarily significant.

The method of trial itself deserves deep investigation. A research project in the case of jury trials was begun at Chicago some years ago and this type of study should be resumed. We do not know if the jury is a more efficient and accurate instrument than the judge alone. We do not even know that the adversary method itself, with its opposing counsel and its ritual of examination and cross-examination, is necessarily a better technique than a modern inquisitorial method, with an impartial investigation of all facts, including witnesses' statements, being entrusted to skilled, competent experts, properly trained in the appropriate areas of the behavioural and other sciences.

The training and qualification of judges is another area that bears scientific investigation. What should a judge's qualifications be, and who should determine them? If examinations as to physical health are now required of candidates for the presidency of the United States, then mental health should logically follow; and if presidents, why not judges? Jerome Frank even suggested the possible psycho-analysis of judges in order to try and eliminate prejudice; similar methods could detect unsuitable candidates. In France judges are trained separately and advance as civil servants up the judicial hierarchy. Is this more defensible than our system, which theoretically chooses the most experienced practitioners
for the Bench, but is often merely a thinly disguised spoils system or patronage technique? If special training or characteristics are required of judges, then we must determine what training and what characteristics. Lest you think I am being unrealistic in asking these questions, let me point out that a number of New York City Bar Associations have recently completed a study in precisely this area, and the Institute of Judicial Administration at New York University has been concerned with this type of problem for many years. But it is time we, in Canada, began to use our psychologists and others and to work with them in these areas. It should be a challenge to the Bar and to our law faculties to immerse themselves in the field of evidentiary and judicial reform, both for the good of society as a whole and for the sake of the legal order in particular.

Conclusion

This article has merely attempted to introduce the reader to some aspects of the new and developing field of jurimetrics. The initial impression that may be given by the symbolism of Boolean algebra is probably one of technical difficulty; this is not the case. The algebra can be learned in a few hours by any high school senior, and its operations are far easier than those of arithmetic. In addition, the coming generation of lawyers will have had some introduction to set theory, perhaps even at the elementary school level, and Boolean algebra is merely one aspect of the theory of sets. Boolean (or two-valued) algebra is the basis of computer theory and design. It should certainly be a prerequisite for legal training in the future as the basis of symbolic logic as well as of computer technology. There is no doubt that a true revolution in legal research and inquiry along these lines has already begun, but whether or not it will reach its full growth in our lifetime is another question.
### Theorems of Boolean Algebra

<table>
<thead>
<tr>
<th>OR: $0 + 0 = 0$</th>
<th>AND: $0.0 = 0$</th>
<th>INVERSION: $\overline{1} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 + 1 = 1$</td>
<td>$0.1 = 0$</td>
<td>$\overline{0} = 1$</td>
</tr>
<tr>
<td>$1 + 0 = 1$</td>
<td>$1.0 = 0$</td>
<td></td>
</tr>
<tr>
<td>$1 + 1 = 1$</td>
<td>$1.1 = 1$</td>
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</tr>
</tbody>
</table>

**SUM FORM**

1. $A + 0 = A$  
2. $A + 1 = 1$  
3. $A + A = A$  
4. $A + \overline{A} = 1$  
5. $A + B = B + A$  
6. $A + AB = A$  
7. $A + B + C = (A + B) + C$  
8. $AB + A\overline{B} = A$  
9. $AB + AC = A (B + C)$  
10. $(A + B + C + \ldots) = \overline{ABC} \ldots$ (DeMorgan’s Laws)  
11. $\overline{A}B + B = A + B$  
12. $AB + BC + \overline{A}C = AB + \overline{A}C$  

**PRODUCT FORM**

1. $A.1 = A$  
2. $A.0 = 0$  
3. $A.A = A$  
4. $A.\overline{A} = 0$  
5. $AB = BA$  
6. ABC = (AB) C  
7. $(A + B)(A + \overline{B}) = A$  
8. $(A + B)(A + C) = A + BC$  
9. $(A + B)(A + C) = A + BC$  
10. $ABC \ldots = \overline{A} + \overline{B} + \overline{C} + \ldots$ (10a)  
11. $(A + \overline{B})B = AB$  
12. $(A + B)(B + C)(\overline{A} + C) = (A + B)(\overline{A} + C)$  

### Symbols

**Constants**

- 0 (False, $F$, ¬
- 1 (True, $T$, +)

**Operations**

- **AND** (conjunction) $XY (X.Y,X\cap Y, X\wedge Y)$
- **OR** (disjunction) $X + Y (XUY,XvY)$
- **INVERSION** (negation, complementation) $\overline{X} (X', \sim X)$
APPENDIX "B"

\[ X = \overline{AC} + BD + \overline{CD} \]
\[ \overline{X} = \overline{AC} + BD + \overline{CD} \]
\[ = \overline{AC} \cdot BD \cdot CD \]
\[ = (\overline{A} + \overline{C})(\overline{B} + \overline{D})(\overline{C} + \overline{D}) \]
\[ = (A + C)(\overline{B} + D)(C + D) \]
\[ = (AB + AD + BC + CD)(C + D) \]
\[ = ABC + ACD + BC + CD + ABD + ADD + BC + CDD \]
\[ = ABC + ACD + BC + CD + ABD + BCD \]
\[ = BC + CD + ABD \]

The same result can be obtained more quickly by using standard minimization techniques, such as the Karnaugh Map or Veitch Diagram.